red of the first series of colors. The red of the fourth ring was faint and seemed to be fringed with white.

The time of observation was so short that I can give nothing more in the way of facts. But from the general impression received I think that if time had allowed I could have made out not only the primary colors but also the mixture as given by Newton for his 1, 2, 3, 4 orders. The three first of these he actually observed in June, 1692, and calculated the rest.

STUDIES ON THE CIRCULATION OF THE ATMOSPHERES OF THE SUN AND OF THE EARTH.

By Prof. FRANK H. BIGELOW.

III.—THE PROBLEM OF THE GENERAL CIRCULATION OF THE ATMOSPHERE OF THE EARTH.

THE CANAL THEORY.

In my Cloud Report, Annual Report of the Chief of the Weather Bureau, 1898–1899, Volume II, chapter 11, it was shown that for the United States the canal theory of the general circulation of the atmosphere, as worked out by Ferrel and by Oberbeck, does not sufficiently conform to the observations on cloud motions to be a satisfactory solution of the problem. The Report of the International Committee, 1903, by H. H. Hildebrandsson, reached the same conclusions for nearly all parts of the Northern Hemisphere, and, therefore, that canal theory may be finally abandoned. The following paper contains some suggestions on this subject which seem promising, and adapted to laying the foundation for a new development of this branch of theoretical meteorology. The physical facts to be accounted for may be found in the two publications referred to, also in my Papers on the Statics and Kinematics of the Atmosphere in the United States,[22] and they need not be recapitulated in this place.

THE GENERAL EQUATIONS OF MOTION.

Referring to the well-known general equations of motion as summarized in the Weather Bureau Cloud Report, from equation (155) we have

$$(1)\qquad -\frac{1}{\rho}\frac{\partial P}{\partial x}-\frac{\partial V}{\partial x}=\frac{du_1}{dt}.$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial y}-\frac{\partial V}{\partial y}=\frac{dv_1}{dt}.$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial z}-\frac{\partial V}{\partial z}=\frac{dw_1}{dt}.$$

These are transformed into the first form of polar equations (181), these again into the forms (200) and (201) in succession, so that the common integral becomes

$$(2)\qquad \int -\frac{dP}{\rho}=\int\left(\frac{du}{dt}\,\partial x+\frac{dv}{dt}\,\partial y+\frac{dw}{dt}\,\partial z\right)+V-C.$$

The usual method of development proceeds by taking

$$(3)\qquad u=\frac{\partial x}{\partial t},\quad v=\frac{\partial y}{\partial t},\quad w=\frac{\partial z}{\partial t},\qquad \text{so that}$$

$$(4)\qquad \int -\frac{dP}{\rho}=\int(u\,du+v\,dv+w\,dw)+V-C$$
$$=\tfrac{1}{2}\,(u^2+v^2+w^2)+V-C$$
$$=\tfrac{1}{2}\,q^2+V-C.$$

This is the ordinary form of the equation of motion on the rotating earth as given in treatises on hydrodynamics, as in Lamb, p. 22, and Basset, Vol. I, p. 34, and is known as Bernoulli's Theorem. C is not an absolute constant, but is the function of the parameter of a stream line; and in the atmosphere, where the flow takes place in stratified layers having different temperatures and angular momenta, it changes from one stratum to another.

It is also possible to integrate these terms along an arbitrary line, $s=\int ds=\int(dx,dy,dz)$, and in this case the derivative relative to the velocity will give acceleration along ds; that is, we have $\dot q ds$ instead of $q dq$, and under some circumstances this may prove to be an advantageous method. In meteorology this will depend, however, upon whether the one or the other set of terms that are required are most practically observed, as line integrals may be readily computed for either of these systems.

LINE INTEGRALS IN THE ATMOSPHERE.

The principles of the canal theory of circulation have been applied by V. Bjerknes[23] and J. W. Sandström[24] in their papers on circulation, under the form of line integrals around arbitrary closed curves in the atmosphere. Thus, the circulation is expressed by them, with the vertical and horizontal components of the total enclosed curve, as

	Total circulation.		Relative component.	Earth's component.
(5)	C_a	$=$	C	$+\,C_e$
(6)	$\int q_a ds$	$=$	$\int q\, ds$	$+\,2\,\omega_0 S_1$
(7)	$\int \frac{dq_a}{dt} ds$	$=$	$\int \frac{dq}{dt} ds$	$+\,2\,\omega_0 \frac{dS_1}{dt}$
(8)	$-\int \frac{dP}{\rho\,.\,ds}\cdot ds$	$=$	$\int \dot q\, ds$	$+\,\frac{d}{dt}\,.\,2\,\omega_0 \int \tfrac{1}{2}\,\varpi \cos i \cos \theta\, ds + R$
(9)	$-\int \frac{dP}{\rho}$	$=$	$\int \dot q\, ds$	$+\,2\,\omega_0 \cos \theta\,.\,\frac{dS}{dt} + R$

Equation (7) is the time rate of change.

C_a = the line integral of the tangential component of total velocity.

C = the line integral of the relative velocity (tangential.)

C_e = the line integral of the velocity of a point on the moving earth itself (tangential).

(q_a, q, q_e) = the velocities; $(\dot q_a, \dot q, \dot q_e)$ = the accelerations.

R = friction; ω_0 = the angular velocity of the earth.

P = pressure; ρ = density.

i = the angle on the plane of the parallel of latitude that ds makes with the direction of a moving point of the earth.

S_1 = the projection of the closed curve S on the plane of the equator for the polar distance θ.

These integrations involve an accurate knowledge of the pressure, density, and acceleration at numerous points along the chosen closed curve, and this it is very difficult to obtain by practicable observations. The variation of S can be found more readily. Several illustrations are given by the authors in applying the theory to the general circulation of the atmosphere and to the local cyclones and anticyclones, but these illustrations do not seem to conform satisfactorily to the conditions observed in North America, as will be set forth in the other papers of this series and in a full report on the subject.

There arises no question with respect to any of the terms of the equation except the one containing $\frac{dS_1}{dt}$, which appears to be an addition to the usual form of the equation of motion on the rotating earth. As has been shown by V. Bjerknes, if the angle θ can be taken constant for a given relatively small closed curve, we have

$$(10)\qquad 2\,\omega_0\frac{dS_1}{dt}=2\,\omega_0\cos\theta\,\frac{d}{dt.}\int\tfrac{1}{2}\,\varpi\cos i\,ds,$$

where i is the angle that the element ds makes with the parallel of latitude, or the angle between the two radii of an ele-

[22] Monthly Weather Review, Vol. XXX, pp. 13, 80, 117, 163, 250, 304, 347.

[23] Meteorol. Zeitschrift, March, 1900; April, 1900; November, 1900; March, 1902.

[24] Kon. Svens. Vet.—*Ak.* Handlingar, *Bd.* 83, No. 4; Meteorol. Zeitschrift, April, 1902; Vetens. *Ak.* 1902, No. 3.

mentary area, as shown in fig. 14. Hence, for a line integral we have,

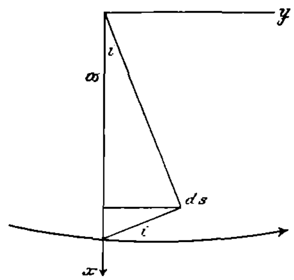

FIG. 14.—Component axes.

$$(11)\quad \frac{d}{dt}\int \tfrac{1}{2}\,\varpi \cos i\,.\,ds = \tfrac{1}{2}\int \frac{d\,\varpi}{dt}\,.\,\cos i\,ds - \tfrac{1}{2}\int \varpi\,\frac{di}{dt}\,.\,\sin i\,ds$$

$$= \tfrac{1}{2}\Big(u\,dy - v\,dx\Big),$$

$$\text{since } \frac{d\varpi}{dt} = u, \quad \varpi\frac{di}{dt} = v, \quad ds\cos i = dy, \quad ds \sin i = dx.$$

We have in the case of a velocity potential, $u\,dy - v\,dx = 0$; and, as is well known, the only influence of the rotation of the earth is to add a deflecting force always at right angles to the direction of motion. The integral of the work done in moving a particle, $\int \frac{dq}{dt}\,.\,ds$, receives no additional term from the fact that the earth rotates, any more than a planet alters the velocity in its orbit from a force perpendicular to its path. We thus obtain $2\,\omega_0\,\frac{dS_1}{dt} = 0$, and all the developments derived from its use must be carefully interpreted. It seems important to have made this fact clear, in order that the equation used as the basis of the following analysis may be taken without modifications. If the gravity potential $V = gz$ is added we obtain the complete equation. The line integral of a gravity force around a closed curve is, also, always zero.

EQUIVALENT EXPRESSIONS FOR THE DENSITY ρ.

The specific volume or isoster, $\frac{1}{\rho} = v$, in the term $\frac{P}{\rho}$, can be discussed in four different ways, and substitutes for it can be introduced into the equation.

1. From Bigelow's equation (47*a*), Cloud Report, we have

$$(12)\qquad \frac{1}{\rho} = \frac{1}{\rho_0}\cdot\frac{P_0}{P}\,\frac{T}{T_0} = \frac{1}{\rho_0}\cdot\frac{P_0}{P}\,(1 + \alpha\,t),$$

where the variations are expressed in terms of ρ_0, P_0, P and the thermometric temperature t. This is the common procedure among meteorologists.

2. From equation (75), the Boyle-Gay Lussac law of gases,

$$(13)\qquad \frac{1}{\rho} = \frac{R\,T}{p} = v,$$

where the variations are given in terms of R, T, p—the gas constant, the absolute temperature, and the weight—and this has been used in some discussions. Since the atmosphere is not arranged upon the adiabatic law, but diverges from it considerably, this method must be cautiously introduced, though there is a strong temptation to use the absolute temperature on account of its convenience.

3. Since we have $\frac{1}{\rho} = \left(\frac{p_0}{p}\right)^{\frac{1}{k}}\frac{1}{\rho_0}$, by equation (84), and $\frac{1}{\rho_0} = \frac{R\,T_0}{p_0}$, by (75), we obtain the third form,

$$(14)\qquad \frac{1}{\rho} = \left(\frac{p_0}{p}\right)^{\frac{1}{k}}\frac{R\,T_0}{p_0},$$

$$(15)\qquad \frac{1}{\rho} = p_0^{\frac{1-k}{k}}\,R\,T_0\,p^{-\frac{1}{k}},$$

where R is the gas constant, and $T_0 = \theta_0$ the potential temperature. This form was employed by H. von Helmholtz, and it has several advantages over the others in applications to the atmosphere.

4. By reducing the volume $\frac{1}{\rho}$ to unit density so that $\rho_0 = 1$, we shall find that

$$(16)\qquad \frac{1}{\rho} = \frac{k}{k-1}\,R^{\frac{1}{k}}\,\theta^{\frac{1}{k}}\,\frac{k-1}{k}\,p^{-\frac{1}{k}},$$

which is the form used by Emden in his paper on the solar circulation.

5. The potential temperature is found practically from the formula

$$(17)\qquad \theta = \theta_0\left(\frac{p}{p_0}\right)^{\frac{k-1}{k}} = \theta_0\left(\frac{B}{B_0}\right)^{0.2889},$$

or in logarithms,

$$(18)\qquad \log\theta = \log\theta_0 + 0.2889\,(\log B - \log B_0).$$

DEVELOPMENT OF THE TERMS $\frac{\omega}{\rho}$, V, AND $\frac{dr}{d\varpi}$.

Since the pressure P in units of force $= g_0 p$, we have from (15)

$$(19)\qquad \frac{P}{\rho} = g_0 p_0^{\frac{1-k}{k}}\,R\,.\,\theta\,.\,p^{-\frac{1}{k}}\,p = g_0 p_0^{\frac{1-k}{k}}\,R\,.\,\theta\,.\,p^{\frac{k-1}{k}}.$$

$$\left.\begin{array}{ll}(20) & \dfrac{P}{\rho} = A\,.\,\theta\,.\,\pi \\ (21) & \dfrac{\partial P}{\rho\partial\varpi} = A\,.\,\theta\,.\,\dfrac{\partial\pi}{\partial\varpi} \\ (22) & \dfrac{\partial P}{\rho\partial r} = A\,.\,\theta\,.\,\dfrac{\partial\pi}{\partial r}\end{array}\right\}\ \text{for}\ \left\{\begin{array}{l} A = g_0 p_0^{\frac{1-k}{k}}\,R\,. = \text{constant.} \\ \theta = \theta_0\left(\dfrac{p}{p_0}\right)^{\frac{1-k}{k}}. \\ \pi = p^{\frac{k-1}{k}} = p^{0.2889}.\end{array}\right.$$

The gravity potential, including the centrifugal force of rotation about the axis z, with the angular velocity ω_0, at the distance ϖ is, for the positive direction of r outwards,

$$(23)\qquad -V = +gr - \tfrac{1}{2}\,\omega_0^{\,2}\,\varpi^2.$$

$$(24)\qquad -V = \frac{g_0 R^2}{r} - \tfrac{1}{2}\,v_0^{\,2}.$$

Hence the original equation (4) is transformed as follows:

$$(25)\qquad \frac{P}{\rho} = -\tfrac{1}{2}\,(u^2 + v^2 + w^2) - V + C.$$

$$(26)\qquad A\theta\pi = -\tfrac{1}{2}\,(u^2 + v^2 + w^2) - \tfrac{1}{2}\,v_0^{\,2} + \frac{g_0 R^2}{r} + C.$$

$$(27)\qquad A\theta\pi = -\tfrac{1}{2}\,(v^2 + v_0^{\,2}) - \tfrac{1}{2}\,(u^2 + w^2) + \frac{g_0 R^2}{r} + C.$$

The equations of motion for two strata flowing over each other, and having different potential temperatures and angular momenta, become,

(28) First stratum:

$$\frac{1}{\theta_1}\,\frac{g_0 R^2}{r} = A\pi_1 + \tfrac{1}{2}\,(v_1^{\,2} + v_0^{\,2})\,\frac{1}{\theta_1} - \frac{C_1}{\theta_1} + \tfrac{1}{2}\,(u^2 + w^2)_1\,\frac{1}{\theta_1}.$$

(29) Second stratum:

$$\frac{1}{\theta^2}\,\frac{g_0 R^2}{r} = A\pi_2 + \tfrac{1}{2}\,(v_2^{\,2} + v_0^{\,2})\,\frac{1}{\theta_2} - \frac{C_2}{\theta_2} + \tfrac{1}{2}\,(u^2 + w^2)_2\,\frac{1}{\theta_2}.$$

At the discontinuous surface of flow the pressure $\pi_1 = \pi_2$, hence,

$$(30)\qquad \left(\frac{1}{\theta_1} - \frac{1}{\theta_2}\right)\frac{g_0 R^2}{r} = \tfrac{1}{2}\,\frac{(v_1^{\,2} + v_0^{\,2})}{\theta_1} - \tfrac{1}{2}\,\frac{(v_2^{\,2} + v_0^{\,2})}{\theta_2}$$
$$- \frac{C_1}{\theta_1} + \frac{C_2}{\theta_2} + \tfrac{1}{2}\,\frac{(u^2 + w^2)_1}{\theta_1} - \tfrac{1}{2}\,\frac{(u^2 + w^2)_2}{\theta_2}.$$

The terms in u and w may not always be neglected where there are strong meridional and vertical currents, as in cyclones and anticyclones.

TO FIND THE DIRECTION OF THE BOUNDARY CURVE BETWEEN TWO STRATA.

1. Differentiate (27) for r with ϖ constant.

$$A\theta d\pi = -\frac{g_0 R^2 dr}{r^2} = -g dr. \tag{31}$$

Then, in crossing the boundary from the first to the second stratum,

$$A\frac{d(\pi_1-\pi_2)}{dr} = -g\left(\frac{1}{\theta_1}-\frac{1}{\theta_2}\right) = -g\left[\frac{\theta_2-\theta_1}{\theta_1\theta_2}\right]. \tag{32}$$

2. Differentiate for ϖ with r constant, at the same time holding the angular momentum ($v\varpi$) constant in each stratum. Equation (27) can be written:

$$g_0\frac{R^2}{r} = A\theta\pi + \tfrac{1}{2}\frac{(v^2\varpi^2)}{\varpi^2} + \tfrac{1}{2}\omega_0^2\varpi^2 + \tfrac{1}{2}(u^2+w^2) - C. \tag{33}$$

Differentiating,

$$0 = A\theta d\pi - \tfrac{1}{2}\cdot\frac{2\varpi(v^2\varpi^2)d\varpi}{\varpi^4} + \tfrac{1}{2}\cdot 2\omega_0^2\varpi d\varpi + u\frac{du}{d\varpi} + \frac{wdw}{d\varpi}. \tag{34}$$

$$A\theta d\pi = +v^2\frac{d\varpi}{\varpi} - v_0^2\frac{d\varpi}{\varpi} - \left(\frac{udu}{d\varpi}+\frac{wdw}{d\varpi}\right). \tag{35}$$

For the two strata,

$$\begin{aligned} A\frac{d(\pi_1-\pi_2)}{d\varpi} &= \frac{1}{\varpi}\left(\frac{v_1^2-v_0^2}{\theta_1}-\frac{v_2^2-v_0^2}{\theta_2}\right) \\ &\quad -\frac{1}{d\varpi}\left[\left(\frac{udu}{d\varpi}+\frac{wdw}{d\varpi}\right)_1\frac{1}{\theta_1}-\left(\frac{udu}{d\varpi}+\frac{wdw}{d\varpi}\right)_2\frac{1}{\theta_2}\right] \\ &= \frac{1}{\varpi}\left[\frac{(v_1^2-v_0^2)\,\theta_2-(v_2^2-v_0^2)\,\theta_1}{\theta_1\theta_2}\right], \end{aligned} \tag{36}$$

omitting terms of the second order.

3. Finally, dividing (36) by (32), we obtain,

$$\frac{dr}{d\varpi} = -\frac{1}{g\varpi}\left[\frac{(v_1^2-v_0^2)\,\theta_2-(v_2^2-v_0^2)\,\theta_1}{\theta_2-\theta_1}\right]. \tag{37}$$

This equation defines the slope of the curve which separates the two stratified currents that flow past each other, preserving their angular momenta, $\Omega = v\varpi = \omega\varpi^2 =$ *constant*, according to the vortex law, where ω is the total angular velocity upon the rotating earth and ϖ is the distance from the axis of rotation. It can be written and interpreted in three different ways, and this gives rise to three cases, each of which finds its application in atmospheric circulations. The equations given in the papers by von Helmholtz and by Emden can be readily transposed into Case I and Case III, but Case II has not been considered heretofore. Omitting terms in u and w, these three cases may be expressed as in equations (38), (39), and (40), following.

CASE I. APPLICABLE TO THE TEMPERATE AND POLAR LATITUDES OF THE EARTH.

$$\theta_1 > \theta_2 \text{ and } \frac{v_1^2-v_0^2}{\theta_1} > \frac{v_2^2-v_0^2}{\theta_2} \text{ for } \begin{bmatrix} v_1 > v_0 \\ v_2 > v_0 \\ v_1 > v_2 \end{bmatrix} \begin{matrix}\text{eastward}\\ \text{relative}\\ \text{velocities.}\end{matrix}$$

$$\frac{+dr}{-d\varpi} = -\left[\frac{(v_2^2-v_0^2)\,\theta_1-(v_1^2-v_0^2)\,\theta_2}{\theta_1-\theta_2}\right] = -\left[\frac{-}{+}\right]. \tag{38}$$

The second member of the equation is positive if

$$\frac{v_1^2-v_0^2}{\theta_1} > \frac{v_2^2-v_0^2}{\theta_2},$$

where $v_1 > v_0$, $v_2 > v_0$, $v_1 > v_2$, and $\theta_1 > \theta_2$, that is to say, if the higher strata have a higher potential temperature and greater eastward relative velocity than the lower, the quantities being arranged as in fig. 15.

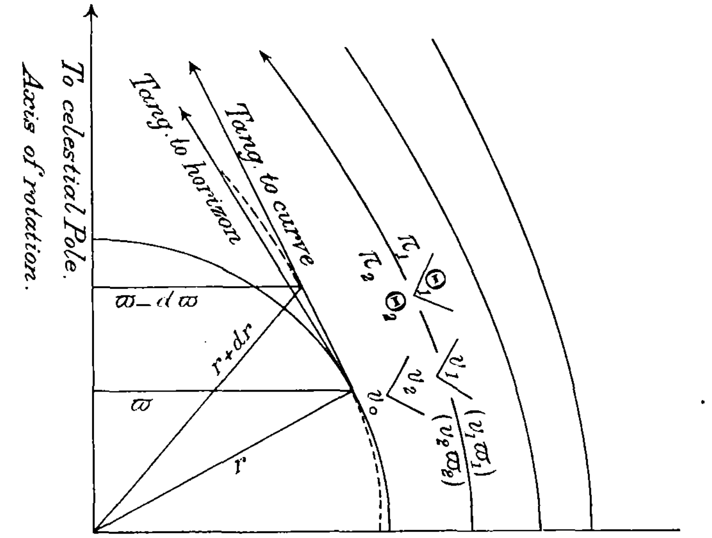

FIG. 15.—Case I.

Take a point in the atmosphere defined by (r, ϖ) the radius and the radius of rotation, respectively. The next successive point on the line of separation of the two gyrating strata is given by $(r+dr)$, $(\varpi-d\varpi)$ as indicated, so that the curve continually rises above the successive tangents to the horizon, but approaches the axis of rotation in the direction of the celestial pole. Since $(v_1^2-v_0^2)$ is the square of the relative linear eastward velocity, it follows that the strata in the atmosphere subject to this law have a continually greater eastward drift and greater potential temperatures with the increase in altitude above the surface. These conditions are characteristic of the earth's atmosphere beyond a certain latitude which varies with the height above the surface. The Weather Bureau Cloud Report, 1898, proved that the velocities and also the potential temperatures for the United States conform to Case I, as in chapters 12, 13, and 14, which contain a discussion of the departure of the temperatures of the upper strata from the adiabatic law in the sense that these strata are overheated. Those velocities have been properly prepared for immediate introduction into the above formula.

CASE II. APPLICABLE TO THE TROPICAL ZONES OF THE EARTH.

$$\theta_1 < \theta_2 \text{ and } \frac{v_1^2-v_0^2}{\theta_1} < \frac{v_2^2-v_0^2}{\theta_2} \text{ for } \begin{bmatrix} v_1 < v_0 \\ v_2 < v_0 \\ v_1 > v_2 \end{bmatrix} \begin{matrix}\text{westward}\\ \text{relative}\\ \text{velocities.}\end{matrix}$$

$$\frac{-dr}{-d\varpi} = -\frac{1}{g\varpi}\left[\frac{(v_2^2-v_0^2)\,\theta_1-(v_1^2-v_0^2)\,\theta_2}{\theta_1-\theta_2}\right] = -\left[\frac{-}{-}\right] \tag{39}$$

The second member of the equation is negative if

$\dfrac{v_2^2-v_0^2}{\theta_2} > \dfrac{v_1^2-v_0^2}{\theta_1}$, where $v_1 < v_0$, $v_2 < v_0$, $v_1 > v_2$, and $\theta_1 < \theta_2$,

that is to say, if the higher strata have lower potential temperatures than the lower, and the lower strata a greater westward relative velocity than the higher, the quantities being arranged as in fig. 16.

Take a point in the atmosphere defined by (r, ϖ) and the next successive point on the line of separation is given by $(r-dr)$, $(\varpi-d\varpi)$, as indicated, so that the curve continually falls below the successive tangents to the horizon, and approaches the axis of rotation in the direction of the celestial pole. The relative velocity is westward, since v_0 is greater than v_1 and v_2, so that $v_1^2-v_0^2$ and $v_2^2-v_0^2$ are both negative quantities. Since $v_1^2-v_0^2$ is a smaller negative quantity than $v_2^2-v_0^2$, the numerator is negative. Also, the denominator is negative, for $\theta_1 < \theta_2$. These conditions are fulfilled in the tropical zones where the westward drift is greater in the lower strata and diminishes upward, while the potential tempera-

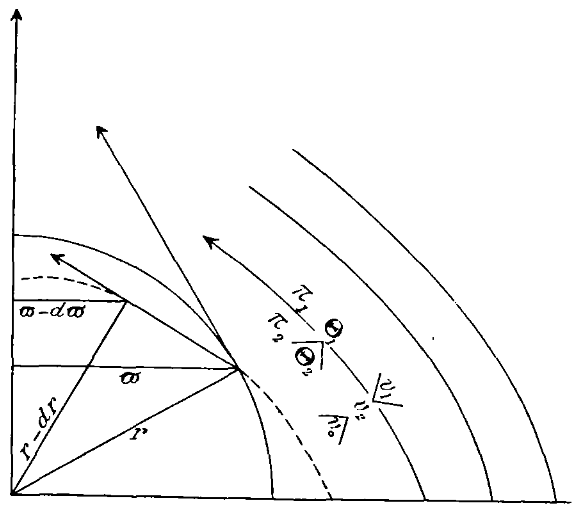

FIG. 16.—Case II.

tures decrease upward. Chapter 8 of the full report will discuss the velocities in the tropical zones of the West Indies. The potential temperatures in the Tropics still remain to be computed.

CASE III. APPLICABLE TO THE ATMOSPHERES OF THE SUN, JUPITER, AND SATURN.

$$\theta_1 > \theta_2 \text{ and } \frac{v_1^2 - v_0^2}{\theta_1} < \frac{v_2^2 - v_0^2}{\theta_2} \text{ for } \begin{bmatrix} v_1 > v_0 \\ v_2 > v_0 \\ v_1 < v_2 \end{bmatrix} \begin{matrix}\text{eastward} \\ \text{relative} \\ \text{velocities.}\end{matrix}$$

$$(40) \quad \frac{+dr}{+d\varpi} = -\frac{1}{g\varpi}\left[\frac{(v_2^2 - v_0^2)\,\theta_1 - (v_1^2 - v_0^2)\,\theta_2}{\theta_1 - \theta_2}\right] = -\left[\frac{+}{+}\right]$$

The second member of the equation is negative if

$\frac{v_2^2 - v_0^2}{\theta_2} > \frac{v_1^2 - v_0^2}{\theta_1}$, where $v_1 > v_0$, $v_2 > v_0$, $v_1 < v_2$, and $\theta_1 > \theta_2$, that is to say, if the higher strata have a higher potential temperature and a smaller eastward relative velocity than the lower, the quantities being arranged as in fig. 17.

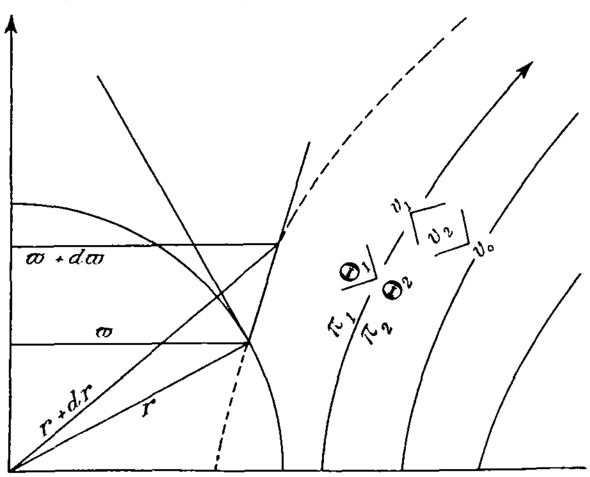

FIG. 17.—Case III.

Take a point in the atmosphere defined by (r, ϖ), and the next successive point on the line of separation, which has varying temperatures but angular momenta that are constant within the thin layers, is given by $(r+dr)\ (\varpi+d\varpi)$, as indicated, so that the curve continually rises above the plane of the horizon, and recedes from the axis of rotation in the direction of the celestial pole. The warmer strata are nearer the axis, and the potential temperature increases in the direction parallel to the axis of rotation, and at the same time the relative velocity is such that the strata near the pole rotate more slowly than those at greater distances. These conditions are found to prevail in the atmospheres of the sun, also of the planets Jupiter and Saturn, as attested by the belt formations and the systems of vortices penetrating to the surface. On the sun the granules of the photosphere are the ends of vortex tubes between adjacent strata having different velocities. Similar vortex tubes are seen on the two planets.

THE INTERACTION OF CASE I AND CASE II IN THE EARTH'S ATMOSPHERE IN THE FORMATION OF LOCAL CYCLONES AND ANTICYCLONES.

In the earth's atmosphere the boundary between the eastward drift of the temperate zones and the westward drift of the tropical zones is an arch spanning the equator high up into the cirrus cloud strata, and resting on the surface at latitudes 30° to 25°. On the poleward side Case I applies but on the side toward the equator Case II prevails.

If the circulations of Case I in the temperate and polar zones, and of Case II in the tropical zones, are applied without further conditions, the isobars in the atmosphere will be distributed, as in fig. 18, so that they rise from the arched boundary of the eastward and the westward relative velocities toward the pole and toward the plane of the equator respectively. This, however, is not the course of the surfaces of pressure in the atmosphere as determined by the observations near sea level, and by computations at higher levels. To illustrate the actual conditions, in fig. 20 Ferrel's values of the isobars on the sea level are given from pole to pole, and Sprung's isobars for the 2000-meter and the 4000-meter planes. The practical problem is, therefore, to account satisfactorily for the modifications of the types. In the present state of meteorology we enter upon a field that is incompletely explored, so that the following remarks are suggestive of the solution rather than final, but there will be much material that sustains them in the complete report, Volume II, Report of the Chief of the Weather Bureau, 1903-1904.

There are two conditions that modify the solutions of Case I and Case II very decisively. (1) The first is that the assumption that the angular momenta in the several strata remain constant around the earth, or that the air rotates in unbroken rings, does not hold good even approximately. Besides the waves and vortices engendered between discontinuous strata, as von Helmholtz explained, there is a yet more powful cause for the breaking down of the vortex law, $v\,\varpi =$ constant, namely, in the cyclones and the anticyclones of middle latitudes, and in the convectional vertical circulation near the equator. (2) The second is that the boundary between the eastward and the westward drift does not girdle the earth uniformly, but is broken up into sections by the intrusion of Case II into the region of Case I, and the extension of Case I into the region of Case II, so that the high pressure belt which this solution assumes to encircle the earth is broken up into large isolated high areas or centers of action, as those lying over the oceans in summer, or over the continents in winter, in the lower strata of the atmosphere. To work out the theory of these details will be a large task for the meteorologist of the future. These two types of disturbance operate together, somewhat as described in the Weather Bureau Cloud Report, 1898-1899, so that the present paper is merely an extension of the analysis there suggested. The following descriptive statement attempts to outline the probable course of the modifications of the pure vortex theory contained in the system of equations given above.

Referring to figs. 18 and 19, the "unmodified" and the "modified" systems, respectively, it is evident that the solar radiation in the Tropics, if unrelieved, will by accumulation raise the isobars of Case II, by increasing the potential temperature θ_2 and the westward velocity $v_2 - v_0$ in the lower strata. In a circulating atmosphere the relief comes in two ways, (1) by forming a vertical convection near the equator, and (2) by forcing a horizontal convection into the lower strata

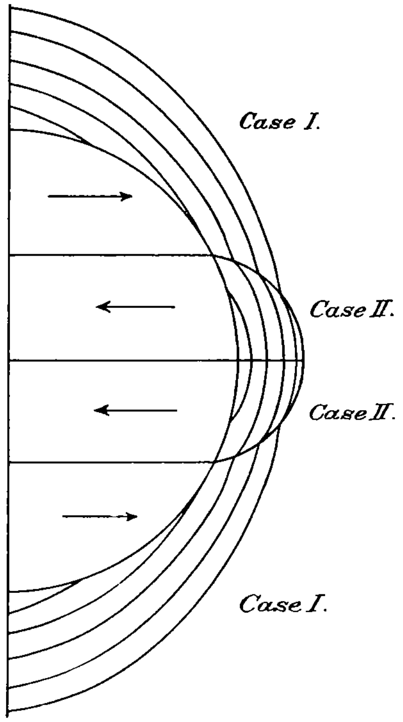

FIG. 18.—Cases I and II unmodified.

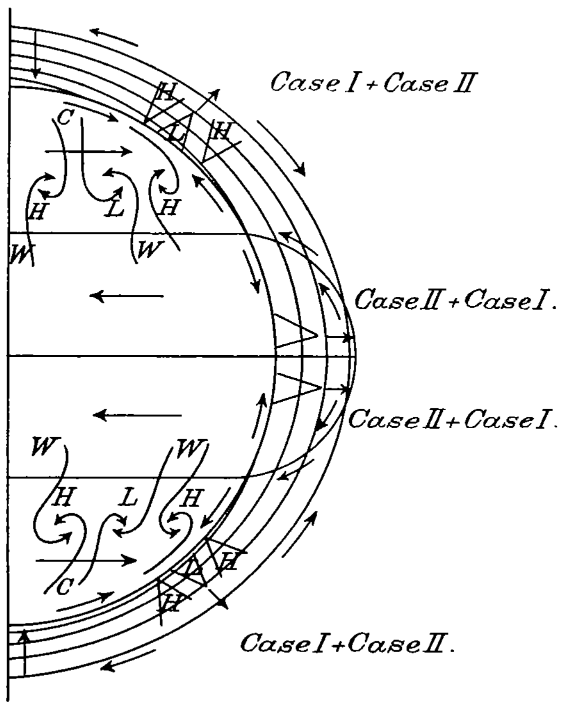

FIG. 19.—Cases I and II as modified.

of the temperate zones. The first transports heat into the upper strata, reducing θ_2 and increasing θ_1, so that the westward drift diminishes. At the same time the intrusion of masses of air having one value of momentum $(mv)_{II}$ into those having another value $(mv)_I$ will change their velocities. These two causes lower the lines of Case II on the equator side, and in the lower strata may even reverse them. Accompanying these changes a component on the meridian toward the equator sets in, so that the trades from the northeast and southeast are developed, and the first minor circulation is maintained in the sense indicated by the arrows over the tropical zone of fig. 19. The rise and fall of the isobars of Case II, with the relief of the incoming solar heat through this circulation, is a complex but sensitive form of natural heat governor which is self-regulating, and preserves the normal state of equilibrium proper for the season of the year. This special action is chiefly due to the mutual movement among the terms of equation (39) for Case II.

A still more complex system relates to the temperate zones and Case I. To some extent the terms within equation (38) for Case I go through a similar self-adjustment in response to the local insolation, but this is by no means the primary cause for the depression of the isobars of fig. 18 to those of fig. 19. As explained in my paper, "The mechanism of countercurrents of different temperatures in cyclones and anticyclones," MONTHLY WEATHER REVIEW, February, 1903, cyclones and anticyclones are formed by horizontal currents underflowing the prevailing eastward drift. Thus, as shown on fig. 19, warm currents flow from the Tropics into the Temperate Zone, as from the Gulf of Mexico into the United States, underneath the eastward drift, and this stratification of warm air beneath cold air produces two changes. The potential temperature θ_2 is increased, the value $\theta_1 - \theta_2$ is diminished, the velocity is checked and the isobars fall, because the angular momentum is diminished. At the same time that the air rises on the east side of the cyclone, a cold current from the north flows to the west side, and this decreases its θ_2 but increases the difference $\theta_1 - \theta_2$, so that the velocities are increased. It is known that the eastern warm current tends to curl westward and the western cold current tends to curl eastward about a cyclonic center; inverted conditions prevail around an anticyclonic center. Furthermore, the dynamic action of intruding cyclones and anticyclones from the lower to the higher strata, by their interchange of inertia with the eastward drift, must diminish the eastward velocity and lower the isobars of Case I. This effect of the interchange of components may be seen by combining the terms of Case I and Case II algebraically. Thus, we have, symbolically,

$$\begin{bmatrix} +dr \\ -d\varpi \end{bmatrix}_{\text{Case I}} + \begin{bmatrix} -dr \\ -d\varpi \end{bmatrix}_{\text{Case II}} = \begin{bmatrix} \text{decrease of } (+dr) \\ \text{increase of } (-d\varpi) \end{bmatrix}$$

so that the lines of Case I are plotted nearer the axis, and lower in the atmosphere above the horizon than in fig. 18. There are instances in which, by this intrusion of the warm air of Case II from the Tropics into the region of Case I, the potential temperature of the lower strata is greater than that of the higher strata, so that Case II supersedes Case I in the temperate zones with local westward winds. Similarly, the interplay of these cases outside their normal regions is a sufficient cause for the manifold local circulations found in the lower strata of the atmosphere up to about 3 miles from the ground, beyond which the circulation is more regular. The amount by which the normal lines of Case I are depressed through the intermixture of Cases I and II, in consequence of temperature and inertia interchanges in the lower strata, measures the amount by which the vortex law ceases to be complete in its application, and by which the Ferrel theory of the general circulation becomes an untenable hypothesis. In effect these interchanges are attended by secondary currents along the meridian so that there is a second minor circuit in the temperate zones, somewhat as indicated on fig. 19. The *H, L, H*, of the vertical section should be understood to stand over *H, L, H*, on the horizontal plane of the given latitude; that is, they are not distributed in latitude but in longitude, and should be superposed in a correct projection. So far as I understand the facts, this circulation, taken in connection with the tropical circuit, conforms to the results of the International Survey, as stated in H. H. Hildebrandson's Report, which need not be here recapitulated. In the polar zone our information is too meager to afford us very definite knowledge, but I suspect that there is a third circuit as shown in fig. 19, though it may not be very pronounced and well defined.

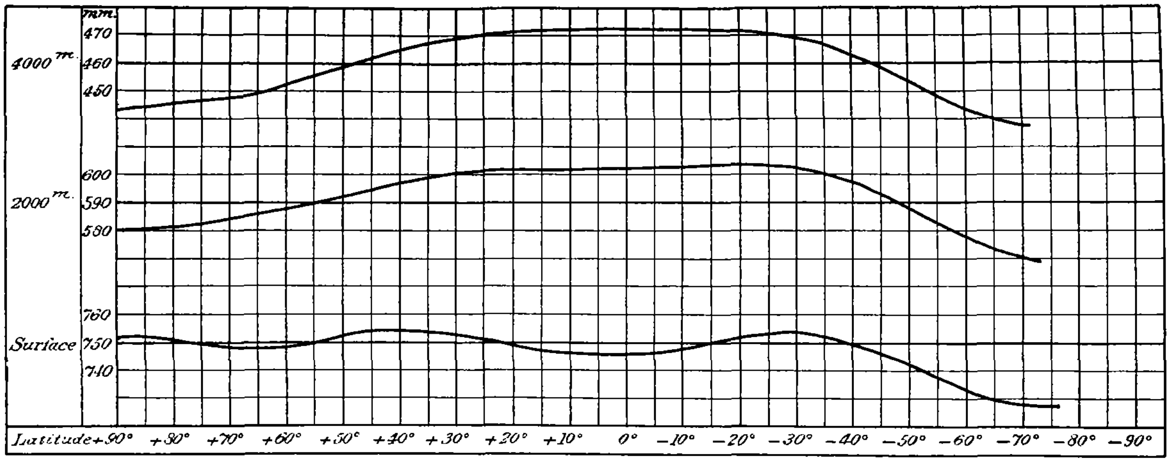

FIG. 20.—Pressures at different latitudes (Ferrel) and altitudes (Sprung).

It is my purpose to work out the data for the temperate and the tropical zones now in the possession of the Weather Bureau and applicable to the North American Continent, along the lines here indicated. The attempt to bring these laws of the general and the local circulations into a harmonious numerical scheme will require considerable labor, but it is believed that it can be accomplished. The data contained in my reports, while apparently somewhat disconnected, are in reality all contributory to my solution of the problems of atmospheric circulations both of the earth and of the sun, together with the connections between them. It is proper to determine carefully the separate portions of the work, i. e., the velocities and temperatures of the strata in motion as dependent upon observations, before trying to put them together in a final synthesis. It is only necessary to have in mind the general plan of development, as here outlined, in order to keep the several portions in harmonious relations with each other.

CLIMATOLOGY OF COSTA RICA.

Communicated by Mr. H. PITTIER, Director, Physical Geographic Institute.

[For tables see the last page of this REVIEW preceding the charts.]

Notes on the weather.—On the Pacific slope, the rainfall was without exception much above the normal. Violent and cold winds have been blowing almost continually, accompanied by mist and rain, which greatly hindered the coffee picking. In San José, pressure temperature and relative humidity were normal, but the rainfall exceeded six times the mean amount for the past fifteen years, 63 millimeters (2.48 inches) and eight days against 10 millimeters (0.39 inches) and three days. Notwithstanding the frequence of rain, the hours of sunshine were above the normal 220.3 against 199.6. The few reports received from the stations of the Atlantic slope indicate a remarkable scarcity of rain in contrast with the diluvial showers of December, 1903.

Notes on earthquakes.—January 14, $2^h\ 37^m$ a. m., slight shock E-W., intensity II, duration 6 seconds; $6^h\ 35^m$ p. m., tremors, apparently E-W., intensity I, duration 3 seconds. January 15, $3^h\ 54^m$ p. m., very slight shock E-W., intensity I, duration 4 seconds; $4^h\ 45^m$ p. m., tremors. January 16, $6^h\ 59^m$ p. m., strong shock E-W., intensity III, duration 2 seconds. January 20, $9^h\ 21^m$ a. m., strong shock E-W., intensity III, duration 6 seconds; $9^h\ 2^m$ p. m., slight shock E-W., intensity I, duration 2 seconds; $9^h\ 18^m$ p. m., shock E-W., intensity II, duration 10 seconds. January 23, $8^h\ 40^m$ p. m., strong shock E-W., intensity III, duration 10 seconds. January 24, $1^h\ 46^m$ a. m., slight shock E-W., intensity II, duration 4 seconds. January 25, $11^h\ 17^m$ p. m., slight shock E-W., intensity I, duration 3 seconds. January 31, $10^h\ 43^m$ p. m., slight shock ENE-WSW., intensity II, duration 3 seconds.

ANNUAL CLIMATOLOGICAL SUMMARY FOR HAWAII.

By R. C. LYDECKER, Territorial Meteorologist.

The following is the rainfall for the year 1903 as gaged at the several stations of the Weather Bureau. The heaviest rainfall during the year was at Nahiku, Maui, at an elevation of 1600 feet. The rainfall here was 319.80 inches, or practically 26.6 feet. The next heaviest rainfall was at Puuohua, Hawaii, at an elevation of 1050 feet, 244.20 inches, or upwards of 20 feet.

Least rainfall, U. S. Magnetic Station, Sisal, Oahu, 8.19 inches.

Approximate percentage of district rainfall as compared with normal: Hawaii, Hilo district, 100 per cent; Hamakua, 110; Kohala, 98; Waimea, 86; Kona, 95; Kau, 62; Puna, 89; island of Maui, 130; island of Oahu, Honolulu district, 72; Nuuanu, 96; Koolau, 67; Ewa, 60; island of Kauai, 72.

Stations.	Elevation.	Amount.	Stations.	Elevation.	Amount.
HAWAII.					
HILO, e. and ne.	*Feet.*	*Inches.*	MAUI.—Cont'd.	*Feet.*	*Inches.*
Waiakea	50	118.89	Puuomalei	1,400	87.40
Hilo (town)	100	132.01	Paia	180	53.76
Kaumana	1,250	174.41	Haleakala Ranch	2,000	60.46
Pepeekeo	100	112.85	Wailuku	250	28.97
Hakalau	200	129.68	OAHU.		
Honohina	300	145.40	Punahou (W. B.), sw	47	32.68
Puuohua	1,050	244.20	Kulaokahua (Castle), sw	50	22.00
Laupahoehoe	500	170.30	Makiki Reservoir	120	32.57
Ookala	400	105.53	U. S. Naval Station, sw	6	18.34
HAMAKUA, ne.			Kapiolani Park, sw	10	14.02
Kukaiau	250	97.29	College Hills	175	38.50
Paauilo	300	75.34	Manoa (Woodlawn Dairy) e	285	95.60
Paauhau	300	60.37	Manoa (Rhodes Gardens)	360	125.95
Honokaa (Mill)	425	68.45	Insane Asylum	30	28.19
Honokaa (Meinicke)	1,100	91.52	Kalihi-uka	485	98.29
Kukuihaele	700	75.04	Nunanu (W. W. Hall), sw	50	35.58
KOHALA, n.			Nuuanu (Wyllie street)	250	54.75
Niulii	200	53.60	Nuuanu (Elec. Station), so.	405	56.08
Kohala (Mission)	521	51.92	Nuuanu (Luakaha), e	850	145.73
Kohala (Sugar Co.)	270	48.45	U. S. Experiment Station	350	45.19
Hawi Mill	700	51.34	Kaliula	1,150	95.08
Puakea Ranch	600	38.07	Tantalus Heights (Frear)	1,360	107.22
Puuhue Ranch	1,847	38.88	Waimanalo, ne	25	28.31
Waimea	2,720	35.00	Maunawili, ne	300	68.42
KONA, w.			Kaneohe	100	41.51
Holualoa	1,350	56.11	Ahuimanu, ne	350	73.59
Kealakekua	1,580	63.22	Kahuku, n	25	19.40
Napoopoo	25	31.25	Wahiawa	900	35.62
Hoopuloa	1,650	45.96	Ewa Plantation, s	60	12.40
Hoopuloa	2,300	65.37	U. S. Magnetic Station	45	8.19
KAU, se.			Waipahu	200	9.30
Kahuku Ranch	1,680	24.42	Moanalua	15	27.85
Honuapo	15	19.32	KAUAI.		
Naalehu	650	29.05	Lihue (Grove Farm), e	200	32.06
Hilea	310	24.97	Lihue (Molokoa), e	300	33.61
Pahala	850	31.60	Lihue (Kukaua), e	1,000	71.19
Volcano House	4,000	67.44	Lihue (Kilohana)	400	38.25
PUNA, e.			Kealia, e	15	16.66
Kapoho	110	72.41	Kilauea Plantation, ne	325	47.34
Pahoa	600	121.10	Hanalei, n	10	80.51
MAUI.			Waiawa	32	10.35
Waiopae Ranch	700	11.95	Eleele	150	20.86
Kaupo (Mokulau) s	285	72.23	Wahiawa (Mountain)	3,000	142.45
Kipahulu	308	80.32	McBryde (Residence)	850	53.41
Nahiku	1,600	319.80	Lawai (Government Road)	450	61.92
Haiku	700	85.40	Lawai, w	225	26.60
Kula (Erehwon)	4,500	35.06	Lawai, e	800	58.98
Kula Waiakoa	2,700	18.70	Koloa	100	29.87